

The statistically anisotropic curvature perturbation generated by $f^2(\phi)F_{\mu\nu}F^{\mu\nu}$

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ABSTRACT: The inflaton might be coupled to a gauge field through a term $f^2(\phi)F_{\mu\nu}F^{\mu\nu}$. If $f \propto a^{-2}$ where $a(t)$ is the scale factor, the perturbation $\delta\mathbf{W}$ of the gauge field generates a potentially observable statistically anisotropic contribution to the primordial curvature perturbation *during* slow-roll inflation. The spectrum and bispectrum of this contribution have been calculated using the in-in formalism of quantum field theory. We give a simpler and more complete calculation using only the classical perturbations. The results suggest that either the entire curvature perturbation ζ (both the statistically isotropic and anisotropic parts) is generated during slow-roll inflation, or else it is generated afterwards.

KEYWORDS: Primordial curvature perturbation.

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1. Introduction

The observed primordial curvature perturbation $\zeta(\mathbf{x})$ presumably originates in the early universe, and a central task of theoretical cosmology is to determine its origin and the evolution $\zeta(\mathbf{x}, t)$. It is usually supposed to be generated from the perturbation of one or more scalar fields, making it statistically isotropic which is consistent with observation. But it may also receive a contribution from one or more vector fields, making it statistically anisotropic. Observation does allow significant anisotropy and in particular a spectrum of the form¹

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k) \left[1 + g_*(\hat{\mathbf{W}} \cdot \hat{\mathbf{k}})^2 \right] \quad (1.1)$$

is allowed with $|g_*| \lesssim 10^{-1}$. The upcoming PLANCK results will give $|g_*| \lesssim 10^{-2}$ barring a detection [1].

¹We use hats to indicate unit vectors

Most schemes invoke a $U(1)$ gauge field B_μ . If the field has the canonical kinetic term and no coupling to gravity, its spectrum increases with wavenumber like k^2 , and its contribution to the spectrum of ζ has the same behaviour making it almost certainly negligible on cosmological scales [3]. One way of avoiding this may be to keep the canonical kinetic term but invoke a coupling to gravity given by $-RB_\mu B^\mu/6$. This apparently [2, 3] generates a flat spectrum but the model has instability and (possibly) non-linearity [4] and it is not known [5] whether or not these spoil the prediction.

In this paper we consider a different scheme, which invokes a gauge kinetic function $f^2(\phi)$ that depends on the slowly rolling inflaton field ϕ . To allow a significant contribution on cosmological scales one needs (at least approximately) $f \propto a^{-2}$ where $a(t)$ is the scale factor, which may be reasonable in the context of string theory [9]. A well-defined contribution to ζ is generated during slow-roll inflation [10, 8] and additional contributions may be generated afterwards [6, 9, 3]. In this paper we give for the first time a master equation that includes all contributions, but our main focus is on the contribution generated during slow-roll inflation. Under the assumption that the homogeneous part of the gauge field dominates the perturbation, it has been calculated [10, 8] using the in-in formalism of quantum field theory. Using instead the classical formula for ζ in terms of the energy density perturbation, we reproduce that calculation and extend it to the case where the homogeneous part is sub-dominant or even negligible. We also identify the assumptions made by the calculation, and show how to perform a more complete calculation with weaker assumptions. We end by discussing the significance of our result.

While this paper was in its final stages of preparation ref. [11] appeared. The authors of this work uses the so called δN formalism to calculate g_* and non-gaussianity in the same model. Our results agree with their findings where there is an overlap.

2. Action and field equations

During inflation we are taking the action to be

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\text{P}}^2 R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right], \quad (2.1)$$

where $F_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ with B_μ a gauge field, and f is some function of the inflaton field ϕ .

Extremizing the action in Eq. (2.1) with respect to fields ϕ , B_μ and their derivatives we obtain field equations

$$[\partial_\mu + \partial_\mu \ln \sqrt{-g}] \partial^\mu \phi + V' + \frac{1}{2} f f' F_{\mu\nu} F^{\mu\nu} = 0; \quad (2.2)$$

$$[\partial_\mu + \partial_\mu \ln \sqrt{-g}] f F^{\mu\nu} = 0, \quad (2.3)$$

where $g \equiv \det(g_{\mu\nu})$, $V' \equiv \partial V/\partial\phi$ and $f' \equiv \partial f/\partial\phi$. We make the gauge choice $B_0 = 0$, which fixes the spatial components $B_i(\mathbf{x}, t)$ up to a constant.

We are going to assume that the inflationary expansion is nearly isotropic, checking later that this is justified. Writing

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \quad (2.4)$$

we then have

$$\ddot{\phi} + 3H\dot{\phi} - a^{-2}\nabla^2\phi + V' = -\frac{1}{2}ff'F_{\mu\nu}F^{\mu\nu}, \quad (2.5)$$

$$\ddot{B}_i + \left(H + 2\frac{\dot{f}}{f}\right)\dot{B}_i - a^{-2}\nabla^2 B_i = a^{-2}2\frac{\partial_j f}{f}\partial_j B_i, \quad (2.6)$$

where $H \equiv \dot{a}/a$ and $\nabla^2 \equiv \delta_{ij}\partial^2/\partial x^i \partial x^j$.

3. Pure slow-roll inflation and the curvature perturbation

If the effect of the gauge field is completely negligible we have pure slow-roll inflation, described for instance in [12, 13]. The last term of Eq. (2.2) is absent, and for the unperturbed inflaton $\phi(t)$ we have

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \quad (3.1)$$

The potential is supposed to satisfy the flatness conditions $\epsilon \ll 1$ and $|\eta| \ll 1$ where $\epsilon \equiv M_{\text{P}}^2(V'/V)^2/2$ and $\eta \equiv M_{\text{P}}^2 V''/V$. Then, more or less independently of the initial condition

$$3H\dot{\phi} \simeq -V'. \quad (3.2)$$

With the flatness conditions this is called the slow-roll approximation, which we use throughout. It gives $|\dot{H}|/H \ll H$ and $|\dot{\epsilon}|/H \ll \epsilon$. Except where stated we take H and ϵ to be constant.

We write $\phi(\mathbf{x}, t) = \phi(t) + \delta\phi(\mathbf{x}, t)$ and define

$$\delta\phi_{\mathbf{k}}(t) \equiv \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}}\delta\phi(\mathbf{x}, t), \quad (3.3)$$

and similarly for other quantities.

Let $N(k)$ be the number of e -folds of slow-roll inflation after the epoch of horizon exit $k = aH$. The ‘cosmological scales’ on which $\zeta_{\mathbf{k}}$ is observed have $N(k_0) - 15 \lesssim N(k) \lesssim N(k_0)$, where $k_0 = a_0 H_0$ and the subscript on the right hand side denotes the present epoch. Also,

$$15 \lesssim N(k_0) \lesssim 70, \quad (3.4)$$

where the upper bound assumes $P \leq \rho$ after inflation, where P is the pressure and ρ the energy density. The lower bound is needed so that cosmological scales leave the horizon. Typical cosmologies give a value in the upper third of the range.

Perturbing Eq. (2.2) to first order, and ignoring the metric perturbation (back-reaction) we have

$$\delta\ddot{\phi}_{\mathbf{k}} + 3H\delta\dot{\phi}_{\mathbf{k}} + (k/a)^2\delta\phi_{\mathbf{k}} = -V''\delta\phi_{\mathbf{k}}. \quad (3.5)$$

Evaluating the metric perturbation to first order, on the flat slicing of spacetime (such that the 3-curvature scalar vanishes) one finds [14, 13] that it is significant only after horizon exit, when Eq. (3.5) becomes

$$\delta\ddot{\phi} + 3H\delta\dot{\phi}_{\mathbf{k}} \simeq -V''\delta\phi_{\mathbf{k}} - 6H^2\epsilon\delta\phi_{\mathbf{k}}. \quad (3.6)$$

The last term represents effect of the metric perturbation (back-reaction). From Eq. (2.2) this term can be written

$$a^{-3}\partial_0\delta(\ln\sqrt{-g})a^3\dot{\phi} \simeq 3H\delta(\ln\sqrt{-g})\dot{\phi}. \quad (3.7)$$

Comparing Eqs. (3.6) and (3.7) we find²

$$\delta(\ln\sqrt{-g}) \simeq \frac{-\sqrt{2\epsilon}}{M_{\text{P}}}\delta\phi_{\mathbf{k}}. \quad (3.8)$$

Regarding $\delta\phi_{\mathbf{k}}(t)$ as an operator its mode function $\phi(k, t)$ satisfies Eq. (3.5). Well before horizon exit at $k = aH$ Eqs. (2.1) and (3.2) describe a free field in flat spacetime. We choose

$$a(t)\phi(k, t) = e^{-ik/aH}/\sqrt{2k}, \quad (3.9)$$

and the vacuum state corresponding to the absence of ϕ particles. Then we have

$$\langle\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k}'}\rangle = (2\pi)^3\delta^3(\mathbf{k} + \mathbf{k}')(2\pi^2/k^3)\mathcal{P}_\phi(k, t), \quad (3.10)$$

$$(2\pi^2/k^3)\mathcal{P}_\phi(k, t) = \phi^2(k, t). \quad (3.11)$$

Also,

$$\langle\delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_2}\delta\phi_{\mathbf{k}_3}\rangle = 0, \quad (3.12)$$

$$\begin{aligned} \langle\delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_2}\delta\phi_{\mathbf{k}_3}\delta\phi_{\mathbf{k}_4}\rangle &= \langle\delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_2}\rangle\langle\delta\phi_{\mathbf{k}_3}\delta\phi_{\mathbf{k}_4}\rangle + \\ &+ \langle\delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_3}\rangle\langle\delta\phi_{\mathbf{k}_4}\delta\phi_{\mathbf{k}_2}\rangle + \langle\delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_4}\rangle\langle\delta\phi_{\mathbf{k}_3}\delta\phi_{\mathbf{k}_2}\rangle, \end{aligned} \quad (3.13)$$

and similarly for higher products.

Well after horizon exit ($k \ll aH \ll 1$) the phase of $\phi(k, t)$ becomes constant which means that $\delta\phi$ can be regarded as a classical perturbation with the above correlators (gaussian perturbation). In the initial regime

$$|\eta|, \epsilon \ll k/aH \ll 1, \quad (3.14)$$

²We assume $\dot{\phi} < 0$ as is the case for hybrid inflation.

the left hand side of Eq. (3.5) is still close to zero, and the mode function is

$$\phi(k, t) \simeq \phi_0(k, t) \equiv \frac{e^{-ik/aH}}{a\sqrt{2k}} (1 - iaH/k) \simeq \frac{iH}{k\sqrt{2k}}. \quad (3.15)$$

This gives a slowly varying perturbation, $H^{-1}|\delta\dot{\phi}_{\mathbf{k}}| \ll |\delta\phi_{\mathbf{k}}|$ with

$$\mathcal{P}_\phi(k, t) \simeq \mathcal{P}_\phi^0 \equiv \left(\frac{H}{2\pi}\right)^2. \quad (3.16)$$

The subsequent evolution is given by Eq. (3.6). It has two independent solutions but the initial slow variation will pick out the solution

$$3H\delta\dot{\phi}_{\mathbf{k}} \simeq -[V'' + 6H^2\epsilon] \delta\phi_{\mathbf{k}}. \quad (3.17)$$

Going to second order in the perturbations of the field and metric one finds additional correlators of $\delta\phi$ (non-gaussianity) [15], starting with the three-point correlator which defines the bispectrum B_ϕ :

$$\langle \delta\phi_{\mathbf{k}_1} \delta\phi_{\mathbf{k}_2} \delta\phi_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3). \quad (3.18)$$

To define the primordial curvature perturbation $\zeta(\mathbf{x}, t)$, one smoothes the metric and the energy-momentum tensor on a super-horizon scale and chooses the comoving threading and the uniform energy-density slicing of spacetime. Then

$$\zeta(\mathbf{x}, t) = \delta \ln a(\mathbf{x}, t) = \delta [\ln a(\mathbf{x}, t)/a(t_1)] \equiv \delta[N(\mathbf{x}, t, t_1)], \quad (3.19)$$

where $a(\mathbf{x}, t)$ is the local scale factor such that a comoving volume element is proportional to a^3 , and $a(t_1)$ is its unperturbed value which can be evaluated at any epoch. As we discuss in Section 6.5 it is usually enough to work to first order in ζ ; then one can choose $t_1 = t$ to get

$$\zeta(\mathbf{x}, t) = H\delta t(\mathbf{x}, t) = -H\delta\rho(\mathbf{x}, t)/\dot{\rho}(t), \quad (3.20)$$

where δt is the displacement of the uniform- ρ slice from the flat slice and $\delta\rho$ is evaluated on the flat slice and

$$\dot{\rho} = -3H(\rho + P). \quad (3.21)$$

Since ρ is smoothed on a super-horizon scale, Eq. (3.21) applies at each location, which implies that $\zeta(\mathbf{x}, t)$ is constant during any era when $P(\mathbf{x}, t)$ is a unique function of $\rho(\mathbf{x}, t)$. Galaxy surveys and observation of the CMB anisotropy detect the value of ζ at the epoch with temperature $T \sim 10^{-1}$ MeV; the universe is then radiation-dominated ($P = \rho/3$) giving ζ a constant value that we denote simply by $\zeta(\mathbf{x})$. Its spectrum is nearly scale-independent with [17]³

$$\mathcal{P}_\zeta(k) \simeq (5 \times 10^{-5})^2 \quad (3.22)$$

$$n(k) - 1 \equiv d \ln \mathcal{P}_\zeta(k) / d \ln k \simeq -0.029 \pm 0.010. \quad (3.23)$$

³The spectrum and bispectrum of any perturbation are defined as in Eqs. (3.10) and (3.18).

The result for $n(k)$ assumes that it has negligible scale dependence. It also assumes a tensor fraction $r \ll 10^{-1}$, which will soon be tested by PLANCK [18].)

For the reduced bispectrum defined by

$$f_{\text{NL}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \equiv \frac{5}{6} \frac{B_\zeta}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_4)} \quad (3.24)$$

where $P_\zeta(k) \equiv (2\pi^2/k^3)\mathcal{P}_\zeta$, current observation [7] give $|f_{\text{NL}}| \lesssim 100$ and barring a detection PLANCK will give $|f_{\text{NL}}| \lesssim 10$. For f_{NL} to ever be observable we need $|f_{\text{NL}}| \gtrsim 1$.

During pure slow-roll inflation, ζ has a time-independent value that we denote by ζ_ϕ . It is well-approximated by Eq. (3.20) with

$$\rho_\phi \equiv V(\phi) + \frac{1}{2}\dot{\phi}^2 \simeq V(\phi), \quad (3.25)$$

leading to the gaussian perturbation $\zeta = \zeta_\phi \equiv -H\delta\phi/\dot{\phi}$ and

$$\mathcal{P}_{\zeta_\phi}(k) = \frac{1}{2\epsilon M_{\text{P}}^2} \left(\frac{H}{2\pi} \right)^2, \quad (3.26)$$

where ϵ and H are evaluated at horizon exit.

Going to higher order in the perturbation of the metric and of $\delta\phi$, one finds [15, 19] a non-zero B_ϕ . Then, using the δN formula to go beyond the first-order formula (3.20), one can use Eqs. (3.10), (3.12), and (3.13) to find $|f_{\text{NL}}| \sim (\mathcal{P}_{\zeta_\phi}/\mathcal{P}_\zeta)^2 10^{-2}$ which is too small to observe [15, 19].⁴

4. Background (unperturbed) universe

Recasting Eqs. (2.5) and (2.6) in terms of \mathbf{W} with components $W_i \equiv fB_i/a$ and dropping gradient terms, one arrives at equations of motion for homogeneous fields $\phi(t)$ and $\mathbf{W}(t)$

$$\ddot{\phi} + 3H\dot{\phi} + V' = \frac{f'}{f} \left| \dot{\mathbf{W}} + \left(H - \frac{\dot{f}}{f} \right) \mathbf{W} \right|^2, \quad (4.1)$$

$$\ddot{\mathbf{W}} + 3H\dot{\mathbf{W}} + \left(2H^2 - H\frac{\dot{f}}{f} - \frac{\ddot{f}}{f} \right) \mathbf{W} = 0. \quad (4.2)$$

We assume $f \propto a^{-2}$ so that the last term of Eq. (4.2) vanishes and there is a solution with $\dot{\mathbf{W}} = 0$. The other solution is decaying, $\mathbf{W} \propto a^{-3}$, and corresponds to $B_i = \text{constant}$. With the action (2.1) this can be set to zero by a gauge transformation and has no physical effect. We therefore take $\mathbf{W}(t)$ to be constant. (For hybrid inflation with the waterfall coupling to \mathbf{W} both solutions are physical [9] but we

⁴This result was first found in [16] by a different method.

assume that the decaying solution is anyhow negligible.) It would make no essential difference if we allow $f \propto a^\alpha$ with α slightly different from -2 , or more generally if we just assume

$$H\dot{W} \simeq - \left(2H^2 - H\frac{\dot{f}}{f} - \frac{\ddot{f}}{f} \right) \mathbf{W}, \quad H^{-1}|\dot{\mathbf{W}}| \ll W \equiv |\mathbf{W}|. \quad (4.3)$$

We assume that the right hand side of Eq. (4.1) is small enough that the approximation $3H\dot{\phi} \simeq -V'$ still applies. Then $f'/f \simeq 2/\sqrt{2\epsilon}M_{\text{P}}$ and treating the right hand side as a first order perturbation we have

$$3H\dot{\phi} \simeq -V' + \frac{18H^2W^2}{\sqrt{2\epsilon}M_{\text{P}}} \simeq -V'. \quad (4.4)$$

The energy density of the gauge field is [20] $\rho_W(\mathbf{x}, t) = -f^2 F_{\mu\nu} F^{\mu\nu}/4$. Smoothed on a super-horizon scale this gives

$$\rho_W = \frac{9}{2} H^2 W^2. \quad (4.5)$$

Using this result for the unperturbed energy density, we see that our assumption that the right hand side of Eq. (4.4) is dominated by the first term is consistent if

$$\frac{2\rho_W}{\epsilon\rho} \equiv \frac{3W^2}{\epsilon M_{\text{P}}^2} \ll 1, \quad (4.6)$$

which we assume.

5. Perturbed universe

Evaluating Eqs. (2.5) and (2.6) to first order in $\delta\phi$ and dropping the right hand side of Eq. (2.6) we get

$$\begin{aligned} \delta\ddot{\phi} + 3H\delta\dot{\phi} - \nabla^2\delta\phi = & -V''\delta\phi + \frac{2}{\sqrt{2\epsilon}M_{\text{P}}} \delta \left[\left| \dot{\mathbf{W}}_{\mathbf{x}} + 3H\mathbf{W}_{\mathbf{x}} \right|^2 \right] - \\ & - \frac{8H\mathbf{W}_{\mathbf{x}}}{\sqrt{2\epsilon}M_{\text{P}}} \left(\dot{\mathbf{W}}_{\mathbf{x}} + 3H\mathbf{W}_{\mathbf{x}} \right) \frac{\delta\dot{\phi}}{\dot{\phi}} \end{aligned} \quad (5.1)$$

$$\delta\ddot{\mathbf{W}} + 3H\delta\dot{\mathbf{W}} - \nabla^2\delta\mathbf{W} = -\frac{2\mathbf{W}_{\mathbf{x}}}{\sqrt{2\epsilon}M_{\text{P}}} \left[\delta\ddot{\phi} - 3H\delta\dot{\phi} - \nabla^2\delta\phi \right] \quad (5.2)$$

$$\mathbf{W}_{\mathbf{x}} \equiv \mathbf{W}(\mathbf{x}, t) \equiv \mathbf{W} + \delta\mathbf{W}(\mathbf{x}, t), \quad (5.3)$$

where \mathbf{W} is the unperturbed value.

This ignores the metric perturbation (back-reaction). Since slow-roll inflation is supposed to be a good approximation, the effect of the metric perturbation in

Eqs. (2.2) and (2.3) is significant only well after horizon exit, and is then given by Eq. (3.8). Including it, Eqs. (5.1) and (5.2) become well after horizon exit

$$\begin{aligned} \delta\ddot{\phi} + 3H\delta\dot{\phi} &= - (V'' + 6H^2\epsilon) \delta\phi + \\ &+ \frac{2}{\sqrt{2\epsilon}M_{\text{P}}} \delta \left[\left| \dot{\mathbf{W}}_{\mathbf{x}} + 3H\mathbf{W}_{\mathbf{x}} \right|^2 \right] - \frac{8H\mathbf{W}_{\mathbf{x}}}{\sqrt{2\epsilon}M_{\text{P}}} \left(\dot{\mathbf{W}}_{\mathbf{x}} + 3H\mathbf{W}_{\mathbf{x}} \right) \frac{\delta\dot{\phi}}{\dot{\phi}} \end{aligned} \quad (5.4)$$

$$\delta\ddot{\mathbf{W}}_{\mathbf{x}} + 3H\delta\dot{\mathbf{W}}_{\mathbf{x}} = - \frac{2\mathbf{W}_{\mathbf{x}}}{\sqrt{2\epsilon}M_{\text{P}}} \left[\delta\ddot{\phi} - 3H\delta\dot{\phi} + \frac{3}{2}H\epsilon\delta\phi \right], \quad (5.5)$$

where in Eq. (5.4) the back-reaction $-6H^2\epsilon\delta\phi$ is the same as in Eq. (3.17).

Let us first set the right hand sides of Eqs. (5.1) and (5.2) to zero. We saw in Section 4.4 how the vacuum fluctuation of $\delta\phi$ is then converted at horizon exit to a nearly gaussian classical perturbation, and the same thing happens to the vacuum fluctuation of $\mathbf{W}_{\mathbf{k}}$ [3].⁵ Its left- and right-handed components have the same mode function $W(k, t) = \phi_0(k, t)$ in Eq. (3.15), which gives well after horizon exit

$$\langle \delta W_{\mathbf{k}}^i \delta W_{\mathbf{k}'}^j \rangle = (2\pi)^3 \left(\delta^{ij} - \hat{k}^i \hat{k}^j \right) \delta^3(\mathbf{k} + \mathbf{k}') (2\pi^2/k^3) (H/2\pi)^2. \quad (5.6)$$

Now we consider the effect of the right hand sides of Eqs. (5.1) and (5.2). In the regime $k \gg aH$, spacetime curvature is negligible and we deal with field theory in flat spacetime. The theory involves massless gauge bosons and the nearly massless inflaton particles ($|V''| \ll (k/a)^2$ which propagate as nearly free particles (perturbative regime), justifying the initial condition (3.9).

To discuss the subsequent evolution, let us take $\delta\phi(\mathbf{x}, t)$ and $\delta\mathbf{W}(\mathbf{x}, t)$ to include only modes with k in some small interval, so that there is a well-defined epoch of horizon exit. In the (still quantum) regime $k \sim aH$, H is the only relevant scale and we have typical magnitudes

$$\delta\phi \sim \delta\mathbf{W} \sim H, \quad \delta\dot{\phi} \sim \delta\dot{\mathbf{W}} \sim H^2, \quad (5.7)$$

$$\delta\ddot{\phi} \sim \delta\ddot{\mathbf{W}} \sim \nabla^2\delta\phi \sim \nabla^2\delta\mathbf{W} \sim H^3. \quad (5.8)$$

Using these with Eq. (4.6), we see that the right hand sides of Eqs. (5.1) and (5.2) are much smaller than H^3 , and hence have only a small effect.⁶ It is therefore reasonable to assume that the perturbations become classical soon after horizon exit, with Eqs. (3.16) and (5.6) initially a good approximation giving typical values $|\delta\phi| \sim |\delta\mathbf{W}| \sim H$.

⁵In contrast with the case for scalar field perturbations [15], the non-gaussianity of $\delta\mathbf{W}$ has yet to be evaluated, but we will assume that it still has a negligible effect.

⁶The findings of this and the previous paragraph remain valid when the last term of Eq. (5.2) is included, and that term is negligible in the super-horizon regime that we are about to discuss.

The evolution of classical perturbations is described by Eqs. (5.4) and (5.5) with $k = 0$. One of their solutions is

$$3H\delta\dot{\phi} \simeq - (V'' + 6H^2\epsilon) \delta\phi + \frac{18H^2}{\sqrt{2\epsilon}M_{\text{P}}} \delta(W^2(\mathbf{x}, t)) \quad (5.9)$$

$$\equiv - (V'' + 6H^2\epsilon) \delta\phi + \frac{18H^2}{\sqrt{2\epsilon}M_{\text{P}}} (2\mathbf{W} \cdot \delta\mathbf{W} + \delta\mathbf{W} \cdot \delta\mathbf{W}) \quad (5.10)$$

$$\delta\dot{\mathbf{W}} \simeq - \frac{2\mathbf{W}}{\sqrt{2\epsilon}M_{\text{P}}} \delta\dot{\phi}, \quad (5.11)$$

which implies $|\delta\dot{\phi}| \ll H|\delta\phi|$ and $|\delta\dot{\mathbf{W}}| \ll H|\delta\mathbf{W}|$. The self-consistency of this solution can be checked by inserting it into the right hand sides of Eqs. (5.10) and (5.11). It is presumably picked out by the initial condition, just as in the case of slow-roll inflation.

Since all of the quantities appearing in the second term of Eq. (5.10) vary slowly on the Hubble timescale, we will take them all to be constant, giving

$$\delta\phi_{\mathbf{k}}(t_{\text{end}}) = \delta\phi_{\mathbf{k}}^0(t_{\text{end}}) + \frac{6N(k)}{\sqrt{2\epsilon}M_{\text{P}}} (\delta(W^2))_{\mathbf{k}}, \quad (5.12)$$

where $\delta\phi_0(t_{\text{end}})$ is the slow-roll result.

6. Contribution of $\delta\mathbf{W}$ to the curvature perturbation

Now we calculate ζ at the end of slow-roll inflation, using Eq. (3.20). Smoothed on a super-horizon scale the energy density is $\rho = \rho_{\phi} + \rho_W$

$$\delta\rho(\mathbf{x}, t_{\text{end}}) \simeq V'\delta\phi(\mathbf{x}, t_{\text{end}}) + \frac{9}{2}H^2\delta(W^2) \quad (6.1)$$

$$\simeq V'\delta\phi_0(\mathbf{x}, t_{\text{end}}) + 18N(k)H^2\delta(W^2) + \frac{9}{2}H^2\delta(W^2) \quad (6.2)$$

$$\simeq V'\delta\phi_0(\mathbf{x}, t_{\text{end}}) + 18N(k)H^2\delta(W^2). \quad (6.3)$$

This gives $\zeta_{\mathbf{k}}(t_{\text{end}}) = \zeta_{\mathbf{k}}^{\phi} + \zeta_{\mathbf{k}}^{\text{W}}$, where

$$\begin{aligned} \zeta_{\mathbf{k}}^{\text{W}} &= \frac{1}{2}C(k)\delta(W^2)_{\mathbf{k}} \\ &= C(k) \left[\mathbf{W} \cdot \delta\mathbf{W}_{\mathbf{k}} + \frac{1}{2} [(\delta\mathbf{W})^2]_{\mathbf{k}} \right], \end{aligned} \quad (6.4)$$

with

$$C(k) = \frac{6N(k)}{\epsilon M_{\text{P}}^2}. \quad (6.5)$$

We can now calculate the spectrum and bispectrum of ζ_W using Eq. (5.6) and the analogues of Eqs. (3.12) and (3.13). The result depends on the value of the unperturbed field \mathbf{W} , and in particular on its magnitude W .

6.1 Working in a finite box

An unperturbed quantity is the zero mode of its Fourier expansion, and at this point we need to remember that within the cosmological context that expansion has to be done within some box of finite coordinate size L [21]. Assuming nearly exponential inflation, the box leaves the horizon $N_L(k) \equiv \ln(kL)$ e -folds before the scale k . To avoid making unverifiable assumptions about an era that will never be constrained by observation (and, as we will see, also in to simplify the calculation) one should take $k_0 L$ to be as small as is consistent with the requirement that the periodic boundary condition implied by the use of the Fourier expansion have a negligible effect (minimal box). Demanding say 1% accuracy in the calculation, it should be enough to choose $k_0 L \sim 100$ corresponding to $\ln(k_0 L) \sim 5$. After choosing L one writes for a given quantity $g(\mathbf{x}, t) = g(t) + \delta g(\mathbf{x}, t)$, where the ‘unperturbed’ value $g(t)$ is the average of g within the box.

Expectation values like (3.10) are in general defined with respect to an ensemble of universes, one of which is the observed universe. But under the usual assumption that perturbations originate as a vacuum fluctuation the translation invariance of the vacuum makes them translation invariant. As a result the expectation values can be defined as spatial averages within a single realisation of the ensemble. Thus the $\langle (\delta g)^2 \rangle$ can be defined as the spatial average within the box, $\langle \delta g(\mathbf{x} + \mathbf{X}) \delta g(\mathbf{x}) \rangle$ can be defined as the average with respect to \mathbf{X} and so on [13].

Keeping only classical modes, Eqs. (3.16) and (5.6) give

$$\langle (\delta \phi)^2 \rangle = \int_L^k \mathcal{P}_\phi(k) dk/k = \ln(k_0 L) (H/2\pi)^2 \quad (6.6)$$

$$\langle |\delta \mathbf{W}|^2 \rangle \simeq 2 \ln(k_0 L) (H/2\pi)^2. \quad (6.7)$$

For the minimal box size this corresponds to typical values $|\delta \phi| \sim |\delta \mathbf{W}| \sim H$.

6.2 The value of W

As was discussed in [21], there are two possible viewpoints about the magnitude of an unperturbed field like \mathbf{W} .⁷ One is to regard the magnitude W as a free parameter, that one gets to choose just like the masses and couplings appearing in the action. The other viewpoint is to estimate the likely value of W , assuming that we are at a typical location within a box whose size M is very much bigger than the size L of the minimal box within which the calculations are done.

Adopting the second viewpoint one has to make an assumption about the evolution of the universe long before the observable universe leaves the horizon. The

⁷These apply to any non-inflaton field that acquires a perturbation from its vacuum fluctuation. The inflaton field is an exception because the inflation model and the cosmology determines its value $N(k_0)$ before the end of inflation. We ignore the issue of anthropic selection, assuming that statistical anisotropy (like non-gaussianity [13]) is neither favoured or disfavoured in that respect.

usual assumption is that there is almost exponential inflation, beginning N_M e -folds before the observable universe leaves the horizon with N_M fairly large. One can then estimate the likely value with suitable assumptions about the relevant physics during those e -folds. In our case, let us assume that the dependence $f \propto a^{-2}$ continues to (at least approximately) hold during those e -folds. Then, after smoothing \mathbf{W} on the scale L Eq. (5.6) gives for the average within the exponentially inflated patch

$$\langle |\delta \mathbf{W}|^2 \rangle \simeq 2 \ln(M/L) (H/2\pi)^2 \simeq 2N_L^2(k) (H/2\pi)^2, \quad (6.8)$$

The expected value of W^2 if our location is typical and therefore

$$W^2 = W_M^2 + \ln(M/L) (H/2\pi)^2 > \ln(M/L) (H/2\pi)^2 = N_M(k_0) (H/2\pi)^2, \quad (6.9)$$

where W_M is the average within the inflated patch. We conclude that if we occupy a typical location within an inflated patch that left the horizon many e -folds before the observable universe, then $W \gg H$.

6.3 The case $W \gg H$

If $W \gg H$, the second term in the square bracket of Eq. (6.4) can be treated as a first-order perturbation. Then, assuming that ζ_W gives the only contribution to the anisotropy of \mathcal{P}_ζ , Eq. (39) of [3] gives

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k) \left[1 + g_*(k) \left(\hat{\mathbf{W}} \cdot \hat{\mathbf{k}} \right)^2 \right], \quad (6.10)$$

with

$$g_*(k) = -\mathcal{P}_{\zeta_W}(k)/\mathcal{P}_\zeta(k) \quad (6.11)$$

$$= -\frac{C^2(k)W^2}{\mathcal{P}_\zeta(k)} \left(\frac{H}{2\pi} \right)^2 \quad (6.12)$$

$$= -48N^2(k) \frac{\rho_W}{\epsilon\rho} \frac{\mathcal{P}_{\zeta_\phi}}{\mathcal{P}_\zeta(k)}. \quad (6.13)$$

Here, \mathcal{P}_{ζ_ϕ} is given by Eq. (3.26), and is independent of k since we are taking H and ϵ to have negligible time-dependence.

The anisotropy is of the form (1.1) but with a strongly scale-dependent $g_*(k) \propto N(k)$. This has not been compared with observation but the constraint is presumably similar to the scale-independent case, currently $|g_*| \lesssim 10^{-1}$. Another constraint comes from the strong scale-dependence of $\mathcal{P}_{\zeta_W}(k)$, giving a contribution $n(k) \supset -g_*(k)$ to the spectral index. Using Eq. (3.23), this requires barring a cancellation $|g_*(k_0)| \lesssim 0.03$. In any case, one certainly needs $|g_*(k_0)| \ll 1$, which with Eq. (6.13) is stronger than Eq. (4.6), justifying the latter. Our assumption that the anisotropy of the expansion has a negligible effect is also justified, because that

anisotropy is presumably only of order (ρ_W/ρ) , which presumably gives a contribution $(\rho_W/\rho)(\mathcal{P}_{\zeta_\phi}/\mathcal{P}_\zeta(k))$ to g_* which is smaller than the one that we have calculated.

Using $\mathcal{P}_\zeta(k_0) = (5 \times 10^{-5})^2$,

$$g_*(k_0) \simeq -1.3 \times 10^{-3} \left(\frac{N(k)}{60} \right)^2 \left(\frac{\mathcal{P}_{\zeta_\phi}}{\mathcal{P}_\zeta(k_0)} \right)^2 \left(\frac{W}{H} \right)^2. \quad (6.14)$$

If we were to assume $\mathcal{P}_{\zeta_\phi} = \mathcal{P}_\zeta$ as in [8], the observational bound $|g_*| \lesssim 10^{-1}$ leads to two conclusions as those authors notice. First, our assumption $W \gg H$ would be only marginally allowed. Also, from Eq. (6.9), one sees that N_M to be very large if we occupy a typical location.

Now we calculate the contribution of ζ_W to the bispectrum B_ζ on cosmological scales. To simplify the calculation we set $N(k) = N(k_0)$. Taking $\mathcal{P}_\zeta(k)$ to be scale-independent and defining

$$\frac{6}{5} f_{\text{NL}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \equiv \frac{1}{4\pi^4} \frac{B_\zeta}{\mathcal{P}_\zeta^2} \frac{\prod k_i^3}{\sum k_i^3}, \quad (6.15)$$

Eq. (41) of [3] gives

$$f_{\text{NL}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = f_{\text{NL}}(1 + f_{\text{ani}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)), \quad (6.16)$$

where the constant prefactor is given by

$$f_{\text{NL}} = \frac{5}{6} \frac{g_*^2(k_0)}{C(k_0)W^2} \quad (6.17)$$

$$= -10N(k_0)g_*(k_0)(\mathcal{P}_{\zeta_\phi}/\mathcal{P}_\zeta), \quad (6.18)$$

and

$$f_{\text{ani}} = \frac{-(\hat{\mathbf{A}} \cdot \hat{\mathbf{k}}_1)^2 - (\hat{\mathbf{A}} \cdot \hat{\mathbf{k}}_2)^2 + (\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2)(\hat{\mathbf{A}} \cdot \hat{\mathbf{k}}_1)(\hat{\mathbf{A}} \cdot \hat{\mathbf{k}}_2)}{\sum k_i^3/k_3^3} + 2 \text{ perms..} \quad (6.19)$$

This expression has not yet been confronted with data, but the bound is presumably similar to the one that takes $g_*(k)$ to be constant, currently $|f_{\text{NL}}| \lesssim 10^2$.

6.4 The case $W \lesssim H$

Consider now the case $W = 0$. Since there is no preferred direction, ζ_W is statistical isotropic. Using Eq. (40) of [3] we have

$$\frac{\mathcal{P}_{\zeta_W}}{\mathcal{P}_\zeta} = \frac{\mathcal{P}_{\zeta_W}^{\text{loop}}}{\mathcal{P}_\zeta} \equiv \frac{4}{3} C^2(k) \left(\frac{H}{2\pi} \right)^4 \ln(kL)/\mathcal{P}_\zeta \quad (6.20)$$

$$= 3N^2(k)\mathcal{P}_\zeta \ln(kL) \left(\frac{\mathcal{P}_{\zeta_\phi}(k)}{\mathcal{P}_\zeta} \right)^2 \ll 1. \quad (6.21)$$

Using Eq. (5.6) and the analogues of Eqs. (3.12) and (3.13) we find

$$f_{\text{NL}} = f_{\text{NL}}^{\text{loop}} \equiv \frac{2}{3} C^3 \left(\frac{H}{2\pi} \right)^6 \frac{\ln(kL)}{(\mathcal{P}_\zeta)^2} \left[1 + \left(\frac{(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2)^2}{\sum_i k_i^3/k_3^3} + \text{c.p.} \right) \right] \quad (6.22)$$

$$= \frac{3^{1/2}}{4} \left(\frac{\mathcal{P}_{\zeta_W}}{\mathcal{P}_\zeta} \right)^{3/2} \ln^{-2}(kL) \mathcal{P}_\zeta^{-1/2} \left[1 + \left(\frac{(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2)^2}{\sum_i k_i^3/k_3^3} + \text{c.p.} \right) \right] \quad (6.23)$$

$$= \frac{9}{4} N^3 \mathcal{P}_\zeta \left(\frac{\mathcal{P}_{\zeta_\phi}}{\mathcal{P}_\zeta} \right)^3 \ln(kL) \left[1 + \left(\frac{(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2)^2}{\sum_i k_i^3/k_3^3} + \text{c.p.} \right) \right] \ll 1 \quad (6.24)$$

In the general case, \mathcal{P}_{ζ_W} and f_{NL} are the sum of the ‘tree’ and ‘loop’ contributions.

6.5 How to make the calculation more accurate

The calculation that we have presented is sufficiently accurate, unless and until statistical anisotropy is observed. If that does happen a more accurate calculation may be required.

Such a calculation should in principle begin with the generation of the classical perturbations $\delta\phi$ and $\delta\mathbf{W}$ from the vacuum, along the lines of [19]. It may in principle generate significant time-dependence and correlation for these quantities, when they first become classical. But the analysis of Section 3 suggests that it will instead confirm the result obtained there; nearly time-independent and uncorrelated perturbations with the spectra $(H/2\pi)^2$ defined in Eqs. (3.10), (3.16), and (5.6).

After the perturbations become classical, their evolution is given by Eqs. (5.4) and (5.5), which can be solved numerical to determine the correlators of the perturbations at the end of inflation. That allows one to determine $\delta\rho$, and working to first order in ζ we can use Eq. (3.20) to determine \mathcal{P}_{ζ_W} and f_{NL} .

Instead of working with the perturbations, one can use the δN formula [3]

$$\zeta(\mathbf{x}, t) = \zeta_\phi(\mathbf{x}, t) + \zeta_W(\mathbf{x}, t) + \sum_i \frac{1}{2} N_{\phi\phi}(t) [\delta\phi_*(\mathbf{x})]^2 + \frac{1}{2} N_{\phi i} \delta\phi_*(\mathbf{x}) \delta W_i^*(\mathbf{x}) \quad (6.25)$$

$$\zeta_\phi(\mathbf{x}, t) \equiv N_\phi(t) \delta\phi_*(\mathbf{x}), \quad (6.26)$$

$$\zeta_W(\mathbf{x}, t) \equiv \sum_i N_i(t) \delta W_i^*(\mathbf{x}) + \frac{1}{2} \sum_{ij} N_{ij}(t) \delta W_i^*(\mathbf{x}) \delta W_j^*(\mathbf{x}). \quad (6.27)$$

Here N is defined by Eq. (3.19) with t_* the epoch of horizon exit for the scale of interest, and the subscripts on N denote partial derivatives evaluated at the unperturbed point in the field space. But as we now argue, the perturbative approach of the previous paragraph is expected to be adequate, i.e. the first-order Eq. (3.20) is expected to be adequate.

The validity of Eq. (3.20) for ζ_ϕ is a standard result but we need to justify its use for ζ_W . The second order correction is presumably of order ζ_W^2 , corresponding to a tiny fractional correction of order ζ_W that can certainly be ignored for the evaluation of \mathcal{P}_{ζ_W} . To see whether it can be ignored for f_{NL} , let us first pretend that \mathbf{W} is a scalar field. Then, setting $C(k)$ to a constant and assuming that the first term of Eq. (6.4) dominates, Eqs. (6.4) and (6.17) give⁸

$$\zeta_\sigma(\mathbf{x}) = \zeta_g(\mathbf{x}) + \frac{3}{5}f_{\text{NL}} \left(\frac{\mathcal{P}_\zeta}{\mathcal{P}_{\zeta_\sigma}} \right)^2 \zeta_g^2(\mathbf{x}), \quad (6.28)$$

where $\zeta_g \equiv CW\delta W$. The first-order formula will therefore be adequate unless

$$f_{\text{NL}} \lesssim \left(\frac{\mathcal{P}_{\zeta_\sigma}}{\mathcal{P}_\zeta} \right)^2. \quad (6.29)$$

Since we need $|f_{\text{NL}}| \gtrsim 1$ for it to be observable, we conclude that the first-order formula will be adequate for the evaluation of f_{NL} unless $|f_{\text{NL}}| \sim 1$ and ζ_W is the dominant contribution to ζ . Except for the second proviso this is a standard result, that was first recognised in the context of the curvaton scenario [23, 24].

Keeping the vector nature of \mathbf{W} , Eqs. (6.4) and (6.17) give

$$\zeta_\sigma(\mathbf{x}) = \zeta_g(\mathbf{x}) + \frac{3}{5}f_{\text{NL}} \left(\frac{\mathcal{P}_\zeta}{\mathcal{P}_{\zeta_\sigma}} \right)^2 \tilde{\zeta}_g^2(\mathbf{x}) \quad (6.30)$$

$$\zeta_g(\mathbf{x}) = C\mathbf{W} \cdot \delta\mathbf{W}(\mathbf{x}) \quad (6.31)$$

$$\tilde{\zeta}_g(\mathbf{x}) = CW|\delta\mathbf{W}(\mathbf{k})| \quad (6.32)$$

At a typical location, $\tilde{\zeta}_g \sim \zeta_g$, and the previous conclusion about the validity of the first-order formula still applies.

7. Conclusion

Working exclusive with the classical perturbation of \mathbf{W} , we have presented a rather complete calculation of the contribution to ζ_W that is generated during slow-roll inflation. In the regime $W \gg H$ we have reproduced the result of [10, 8] for the spectrum (Eq. (6.13)) and of [8] for the bispectrum (Eqs. (6.16) and (6.18)), discussing for the first time the assumptions that are needed to obtain it.

The field $\mathbf{W}(\mathbf{x}, t)$ may survive after the end of slow-roll inflation, and generate further contributions to ζ . The effect of these is to change $C(k, t)$, so that Eqs. (6.11) and (6.17) as well as Eqs. (6.20), (6.22), and (6.23) still hold but with C the final value.

⁸If instead the second term dominates, it is solely responsible for f_{NL} and the first-order formula will certainly be adequate.

A contribution ζ_{end} may be generated during the waterfall that ends hybrid inflation. It can be calculated from the ‘end-of-inflation’ formula, provided that the waterfall is sufficiently brief which requires $H \lesssim 10^{-9} M_{\text{P}} (\mathcal{P}_{\zeta_{\text{end}}}/\mathcal{P}_{\zeta})^{1/2}$.⁹ The result is

$$C_{\text{end}} = -\frac{h^2}{\sqrt{2\epsilon} M_{\text{P}} m g} \quad (7.1)$$

If the gauge symmetry is spontaneously broken after inflation, the curvaton mechanism may generate another contribution after inflation. With the simplest assumption that \mathbf{W} is nearly time-independent until it begins to oscillate

$$C_{\text{curv}} = \frac{\rho_W^{\text{dec}}}{\rho^{\text{dec}}} \frac{1}{3W^2}, \quad (7.2)$$

where ‘dec’ denotes the epoch just before the decay of \mathbf{W} .

Including both these contributions we have

$$C = C_{\text{sr}} + C_{\text{end}} + C_{\text{curv}}, \quad (7.3)$$

where $C_{\text{sr}} = 6N(k)/\epsilon M_{\text{P}}^2$ is the contribution generated during slow-roll inflation. The extra contributions allow the non-gaussianity to be observable even if $W \ll H$.

We close with an important comment, stemming from Eq. (6.14). On the reasonable assumption that there were some large number N_M of e -folds of inflation before the observable Universe left the horizon, one expects $W/H \sim N_M \gg 1$. But then Eq. (6.14) is compatible with the observational bound on g_* only if $\mathcal{P}_{\zeta_\phi} \ll \mathcal{P}_{\zeta}$; in other words, if the the observed curvature perturbation is mostly generated *after* slow-roll inflation by for instance the end-of-inflation or curvaton mechanism. Our results therefore suggest a dichotomy regarding the generation of observable statistical anisotropy of the curvature perturbation. Either the entire curvature perturbation ζ (both the isotropic and the statistically anisotropic part) is likely to be generated during slow-roll inflation, or it is likely to be generated afterwards.

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⁹This result is an obvious extension of the one that was derived in [22] with the assumption $\mathcal{P}_{\zeta_{\text{end}}} = \mathcal{P}_{\zeta}$. It follows from the fact that the the duration of the waterfall cannot be much less than $1/m$.

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